Vehicle Suspension Inspection by Stewart Robot

M.Kazemi[1,*] and M. Jooshani[2]

[1] Assistant Professor ,Department of Electrical Engineering ,Shahed University, Tehran, Iran[-2] Msc Student Electrical Engineering Faculty, Southern Tehran Islamic Azad University

kazemi@shahed.ac.ir

Abstract

The suspension system of a vehicle is one of the most important parts which is involved in the process of vehicle designing. When a vehicle suspension system is designed, the evaluation of its performance against the road disturbances such as shocks and bumps are very important. The most commonly used systems consist of four hydraulic Jacks with mobility in vertical line with low speed and low exactitude. This paper offers a new mechanism for inspecting the suspension system of a vehicle using a parallel robot called Stewart. This robot is a special kind of parallel robots with capability of movements in different directions with high speed, accuracy and repeatability. In this paper the suspension system is evaluated on a quarter model of a simulated vehicle with control and guidance of Stewart robot using PID controller. The Stewart robot simulates the isolated and uneven bumps on a flat road in order to evaluate the given suspension system, and to investigate some criteria such as comforting of the passengers and remaining of the vehicle on the road. The results of the simulations show that the proposed method has a high accuracy, applicability and flexibility as well as simplicity, compared to currently used mechanisms.

Keywords: *Vehicle suspension, Stewart robot, Quarter model of a vehicle, PID controller.*

1. INTRODUCTION

The suspension system of a vehicle is the part between the cabin of the passengers, chassis and the wheels of the vehicle which adjust the reaction of the cabin and chassis in accordance to the road. The suspension systems were first introduced as an attempt to solve the problem of the shock to the passenger cabin or the cart due to the transmission of forces from the bumps to the wheels and then the passengers [1].

In the following years in 1904 [2], more improvements were made by William Brush in the suspension system and finally in 1906 cars with the modern suspension systems were built [3] and a revolution was created including shock absorbers which were installed on flexible wooden axis. Apart from exceptional cases, this kind of spring was used for 25 years in front suspension system after the Brush's introduction. Then suddenly in 1934 [4], General Motors, Crysler, Hudson and others again started to make new front suspension springs and this time a spring is installed for each wheel individually. Testing and assessment of applicability of each suggested suspension system is a pursuit of different testes which qualifies it. So far various ways have been suggested for testing [5] the suspension system of vehicle [6-7].

In the conventional test for vehicle suspension system, hydrolic jacks are used which have limitations in speed and movement. There are also several testing stages on the parts of suspension system which are performed by manufacturing companies but since these testing stages are done individually on each part, they cannot qualify the whole suspension system. Due to the capabilities of Stewart robots [8], the paper has proposed to use them for suspension system inspecting. By installing four robots at the end of production line and providing them to appropriate related control software, the test and inspection of suspension system can be performed very fast and very exact under different circumstances. The inspection results then can be used to improve the suspension system and producing a high quality suspension system. The simulations are done with the use of SimMechanics from the Simscape toolbox in the MATLAB software.

The paper is organized as follows. The dynamic and kinematic equations of Stewart robot are analyzed

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in Second 2. Section 3 mainly presents the modeling of suspension system. The overall setup and simulation results are presented in Section 4, and finally Section 5 concludes the paper.

2. Stewart Robot equations

There are mainly two types of the manipulators: serial manipulators and parallel manipulators. The serial manipulators are open-ended structures consisting of several links connected in series. Such a manipulator can be operated effectively in the whole volume of its working space. However, as the actuator in the base has to carry and move the whole manipulator with its links and actuators, it is very difficult to realize very fast and highly accurate motions by using such manipulators. As a consequence, there arise the problems of bad stiffness and reduced accuracy. Parallel robots are described as a closed kinematic chain in which the tool supporter is installed on the robot base with many kinematic chains. Due to the hardness and capability of working in high speed and capability of lifting heavy objects of parallel robots, these robots attracted a lot of attention in 1990 in scientific articles and also in industry [8]. Solving the inverse kinematics, i.e. determining the leg lengths once the position and orientation of the top platform are known, is easy to do. Finding the position and orientation of the top platform with the leg lengths known is, however far more complicated.

Furthermore, the closed mechanical chains make the dynamics of parallel manipulators highly complex and the dynamic models of them highly non-linear. So that, while some of the parameters, such as masses, can be determined, the others, particularly the friction coefficients, can't be determined exactly. Because of that, many of the control methods are not efficient satisfactorily. In addition, it is more difficult to investigate the stability of the control methods for such type manipulators [14]. Under these conditions of uncertainty, a way to identify the dynamic model parameters of parallel manipulators is to use adaptive control algorithms, Fuzzy control, intelligence techniques, etc. In some cases the researchers tried to simplify the robot dynamics and with considering some factors and combining the methods based on dynamic modeling, finding a faster and more accurate robot controller which is faster but their time consuming calculations are still a main problem. The dynamics of the parallel robots have a complicated formulation because of their closed loop and kinematic restraints. However, there are a lot of researches that work on the Stewart robot dynamics [9-11].

2.1. Kinematics

In this kind of robot there are a group of mechanical arms which are all connected to one platform in order to be able to gain the feeling of bending or stretching in different directions with high speed and high accuracy. It is a mechanical machine with six jacks which are located in three pairs between two platforms, see Fig. 1. The upper rigid body forming the mobile platform, M , is connected to the lower rigid body forming the fixed base platform, B , by means of six legs. Each leg in that figure has been represented with a spherical joint at each end. Each leg has upper and lower rigid bodies connected with a prismatic joint, which is, in fact, the only active joint of the leg [3].

Motion of the moving platform is generated by actuating the prismatic joints which vary the lengths of the legs, q^{L_i} , $i = 1,...,6$. So, trajectory of the center point of moving platform is adjusted by using these variables.

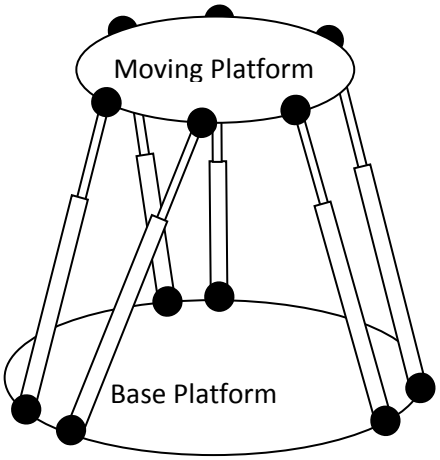

Fig1. A 6-dof Stewart manipulator

For modeling the robot, a base reference frame (O_B, x_B, y_B, z_B) is defined as shown in Fig. 2. A second frame (O_M, x_M, y_M, z_M) is assigned to the center of mobile platform, O_M , and each leg is attached to the base platform at point B_i and to the mobile platform at point Q_i for $i = 1,...,6$. The pose of the center point, O_M , of moving platform is represented by the vector

$$x = \begin{bmatrix} x_B & y_B & z_B & \alpha & \beta & \gamma \end{bmatrix}^T \qquad (1)$$

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where x_B, y_B, z_B are the cartesian positions of the point O_M relative to the base frame and α, β, γ are the rotation angles, namely Euler angles, representing the orientation of mobile frame relative to the base frame by three successive rotations about the x_M, y_M and z_M axes, given by the matrices $R_x(\alpha)$, $R_y(\beta)$, $R_z(\gamma)$ respectively. Thus, the rotation matrix between the base and mobile frames is given as follows:

$$R_B^M = R_x(\alpha)R_y(\beta)R_z(\gamma) \qquad (2)$$

Then the inverse kinematics can be analyzed by the representation of any one of its legs. By using the rotation matrix given by equation (2), the position vector of the upper joint position, Q_i, connecting the mobile platform to the leg i, $q^Q{}_i$ can be transformed to the base frame as follows:

$$q^Q{}_i = p^o + R_B^M d_i \qquad i = 1,...,6 \qquad (3)$$

where p^o represents the position vector of the center point of mobile platform, O_M, relative to the base frame, d_i is the position vector of the point $Q_i, i = 1,...,6$, relative to the mobile frame. Then the vector $q^A{}_i$ representing the leg lengths between the joint points B_i and Q_i can be transformed to the base frame as follows:

$$\vec{B_iQ_i} = q^A{}_i = -a_i + q^Q{}_i \qquad i = 1,...,6 \qquad (4)$$

where a_i represents the position vector of the point B_i, relative to the base frame. The leg lengths $q^A{}_i$, is then obtained by Euclidean norm of the leg vector given above. So, using equation (3) and (4) we can write

$$\left(q^A{}_i\right)^2 = \left(a_i + p^o + R_B^M d_i\right)^T \left(a_i + p^o + R_B^M d_i\right).$$

The leg lengths related to a given pose of mobile platform can be obtained for a trajectory defined by the pose vector x, given in equation (1).

2.2. Dynamics

The Newton-Euler equations of the described Stewart manipulator can be derived in a more compact form as described below [14-15]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \qquad (5)$$

Where, q is the generalized coordinate vector, $\tau \in R^6$ is the generalized force developed by the actuators, and $G(q)$ is the gravity vector. The symmetrical and positive definition matrix $M(q) \in R^{6\times6}$ is determined as:

$$M(q) = T^T M T \qquad (6)$$

where, $M = diag(M_1, M_2, ..., M_{13})$ denotes the mass and moment of inertia properties of the all thirteen rigid bodies in the manipulator, and the generalized wrench vector $T = [t_1^T \; t_2^T \; ... \; t_{13}^T]^T$ is defined in terms of the angular and linear velocities.

$C(q,\dot{q})$ is the coefficient matrix of the vectors of Coriolis and centripetal force as given below.

$$C(q,\dot{q}) = T^T M \dot{T} + T^T \Omega M T \qquad (7)$$

where, $\Omega = diag(\Omega_1, \Omega_2, ..., \Omega_{13})$ denotes the angular velocity of the all thirteen rigid bodies of manipulator.

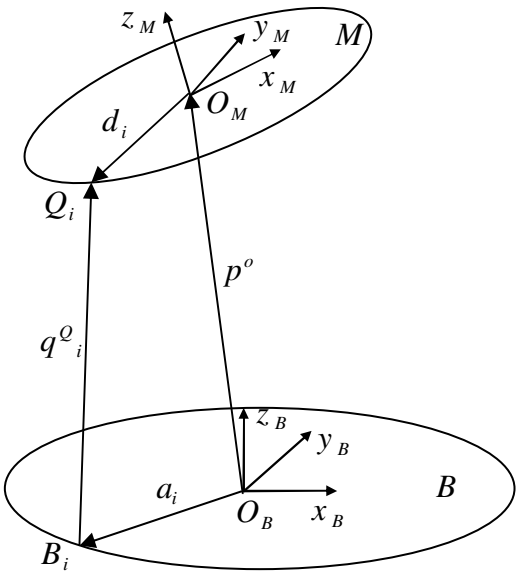

Fig2. Defining the vectors of Stewart robot

3. Suspension System Modeling

For designing and modeling of suspension system there are many ways such as inactive suspension, self adjusting suspension [12], semi-active suspension [5] and active suspension [7]. Since this article is about the suspension test and simulation, and no control is

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supposed to be done on the suspension system, a quarter model of inactive suspension system is used. Active suspension systems require external force or energy to be able to active the control system permanently and control the forces which are transferred through the suspension system.

In this part a second order model of quarter vehicle suspension system (of inactive suspension) is presented. This model has been used in several articles and contains many important characteristics of complicated models of suspension system. Fig. 3 shows the inactive suspension system which is placed on a Stewart robot. The dynamics equations of a passive suspension system for elastic mass and non elastic mass are described as [13]

$$\begin{aligned} &m_s\ddot{z}_s(t)+c_s[\dot{z}_s(t)-\dot{z}_u(t)]+k_s[z_s(t)-z_u(t)]=0 \\ &m_u\ddot{z}_u(t)+c_s[\dot{z}_u(t)-\dot{z}_s(t)]+k_s[z_u(t)-z_s(t)]+ \\ &+k_t[z_u(t)-z_r(t)]+c_t[\dot{z}_u(t)-\dot{z}_r(t)]=0 \end{aligned} \quad (8)$$

where, m_s is the elastic mass (quarter of the mass of chassis mass), m_u is non-elastic mass (the mass of wheel group), c_s and k_s are the damping coefficient and the hardness of spring in passive suspension, c_t and k_t are damping coefficient and contractility of pneumatic tire. $z_s(t)$ and $z_u(t)$ are movement of elastic mass and non-elastic mass and $z_r(t)$ is the movement of Stewart robot to simulate the movement of road as an input.

The state variables is defined as $x(t)=[x_1(t)\ \ x_2(t)\ \ x_3(t)\ \ x_4(t)]^T$,where $x_1(t)=z_s(t)-z_u(t)$ is the deviation of suspension part, $x_2(t)=z_u(t)-z_r(t)$ is the tire deviation, $x_3(t)=\dot{z}_s(t)$ is the velocity of elastic mass and $x_4(t)=\dot{z}_u(t)$ is the velocity of non-elastic mass. Then the state space equation of the system can be expressed as follows.

$$\dot{x}(t)=Ax(t)+Bu(t) \quad (9)$$

where,

$$A=\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ \frac{-k_s}{m_s} & 0 & \frac{-c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{-k_s}{m_u} & \frac{-k_t}{m_u} & \frac{c_s}{m_u} & \frac{-(c_s+c_t)}{m_u} \end{bmatrix}, B=\begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_u} \end{bmatrix}, \quad (10)$$

$$u(t)=\dot{z}_r(t)$$

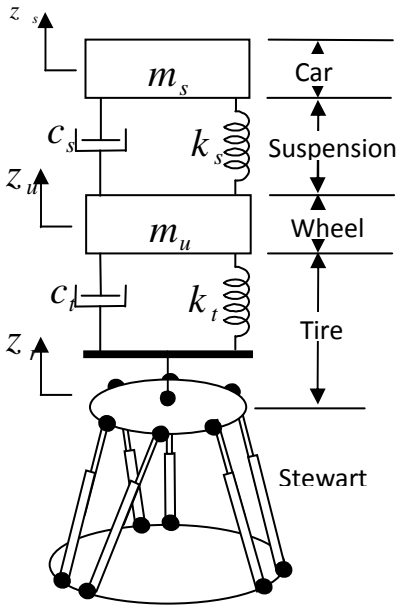

Fig3. Passive model of a quarter car suspension system with Stewart robot.

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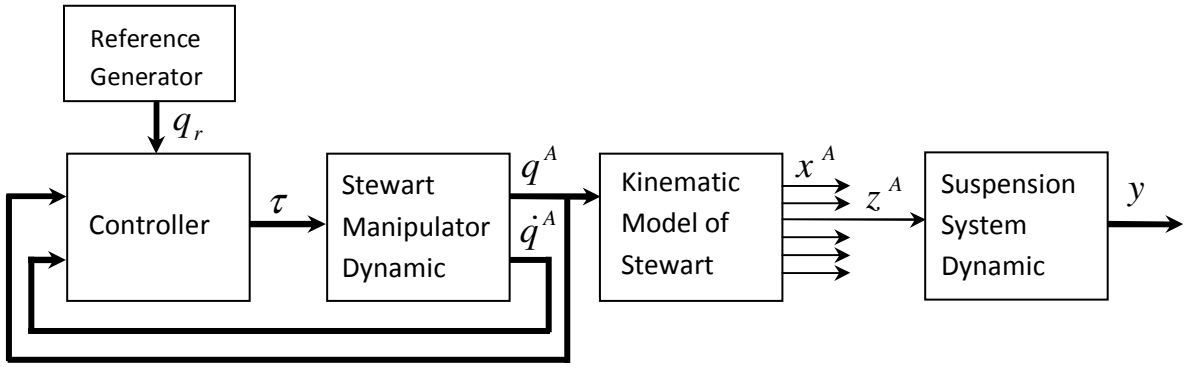

Fig4. Control system configuration

Table 1. Parameters of passive vehicle suspension system for quarter model

parameter	m_s	k_s	c_s	m_u	k_t	c_t
value	320 *Kg*	18 *N/m*	1 *N.s/m*	40 *Kg*	200 *N/m*	0

The system is simulated in SimMechanics toolbox of MATLAB software. The distance between the wheel axis and the mobile platform is considered as 30 centimeter (non contracted position tire). In static balance, both springs shown in Fig. 3 will be contracted. The contraction amount can be calculated through the following equations.

$$k_s \Delta x_s = m_s g \Rightarrow \Delta x_s = \frac{m_s g}{k_s},\ g = 9.81 m/s^2$$
$$k_t \Delta x_t = (m_s + m_u) g \Rightarrow \Delta x_t = \frac{(m_s + m_u) g}{k_t} \quad (11)$$

The mass center in balance situation will be located on the z axis which means $0.3 - \Delta x_t$. The suspension and chassis mass of the car are also defined same as wheel and tie group (a transmission joint and a mass). The natural length of suspension is considered as 60 cm (without any pressure due to vehicle weight).

4. Overall Setup and Simulation Results

Combination of quarter suspension system model to Stewart robot is the next step for creating a unified mechanical model. The base platform of Stewart robots is connected to the ground and the each wheel of vehicle is located on one Stewart robot. When the tire touches the mobile platform, the quarter vehicle suspension system model can be combined to Stewart robot model and a unified mechanical model is achieved.

After achieving the whole robot and suspension system through the relation between the robot support length and its mobile platform, the control commands will be changed into the support length commands. Then control of the mobile platform which is considered as the road surface in this section, will be done by use of a PID controller.

In this way by a closed loop control structure as Fig. 4, the different road circumstances are provided for inspecting the suspension system. A PID controller is considered for the robot movements as follow:

$$G_c(s) = K_p + \frac{K_i}{s} + K_d \frac{Ns}{s + N} \quad (12)$$

Where, N is the filter stability on derivative part which is considered equal to 1000. The controller is tuned for achieving maximum 20% overshoot and permanent error lower than 5% .

In designing a suspension system three criteria are usually taken into consideration; (1): passenger comfort, (2): limitation of deviation in suspension part, (3): ability to remain on the road. Acceleration of the elastic mass can be treated as comfort of the passenger, i.e. defining the first output of suspension system, $y_1(t) = \ddot{z}_s(t)$, as a criteria for comforting of passenger. In order to prevent any harm to the

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passenger and satisfying the comforting of travelers, deviation oscillation of the vehicle suspension part should be decreased. So the following constraint can be considered for the suspension system,

$$|x_1(t)| = |z_s(t) - z_u(t)| \le z_{\max} \quad (13)$$

where z_{max} is the maximum acceptable deviation of suspension part in different road conditions. To have a continuous contact between the tire and the road, it is required that the static load of the tire be greater than its dynamic load, i.e.

$$k_t[z_u(t) - z_r(t)] < 9.8(m_s + m_u) \quad (14)$$

At first we analyze the closed loop system (the combination of Stewart robot and suspension system with the PID controller) with its step response. At time $t_0 = 0.5$ sec. a movement about 10 cm in the direction of z axis, is commanded to Stewart robot to simulate the road displacement. The top plate movement of the robot is shown in Fig. 5. Figures 6 and 7 show the displacement and acceleration of chassis mass center respectively. Deviation of suspension part, wheel displacement, and movement of the center of wheel group mass center, are shown in the figures 8 to 10 respectively. In general a good suspension system should be able to absorb the force which is imposed to the car by the road bumps and to damp it slightly.

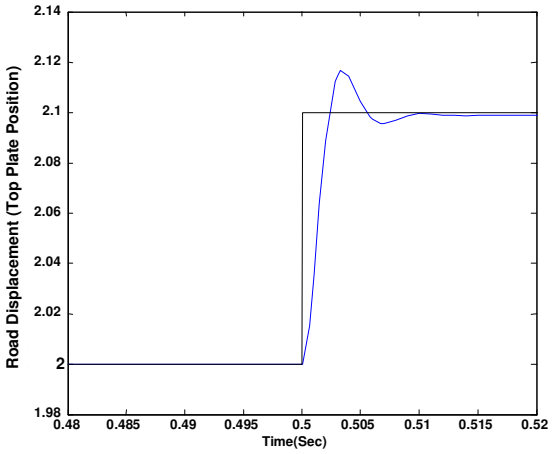

Fig5. Step response of the Stewart robot

For testing and evaluating the suggested design, isolated bump and uneven bump on flat surface roads are simulated by the robot and then comforting of the passenger and remaining the vehicle on the road, are investigated.

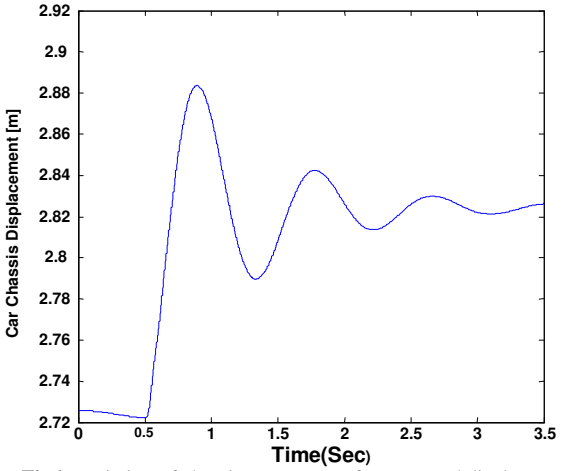

Fig6. Deviation of chassis mass center for step road displacement

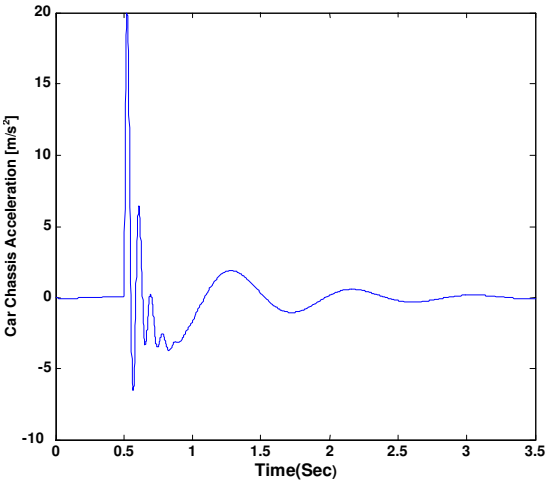

Fig7. The acceleration of the car chassis mass center for step road displacement

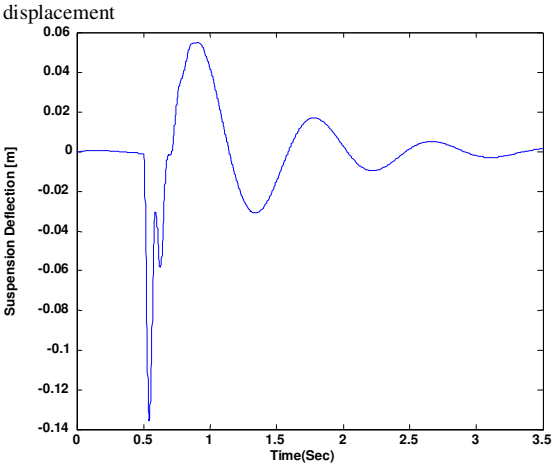

Fig8. Deviation of suspension part response for step road displacement

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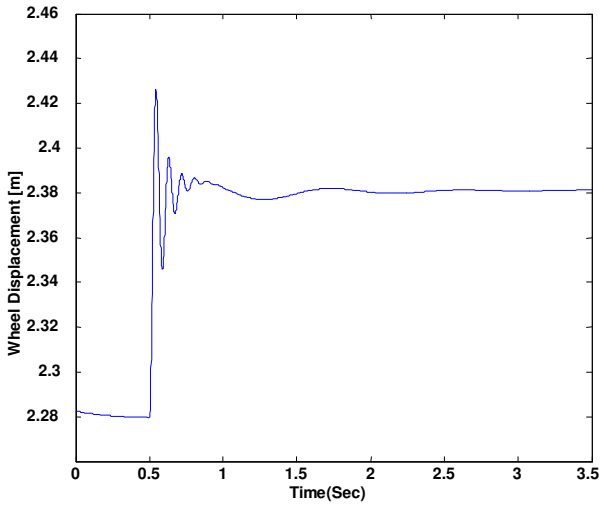

Fig9. Wheel deviation for step road displacement

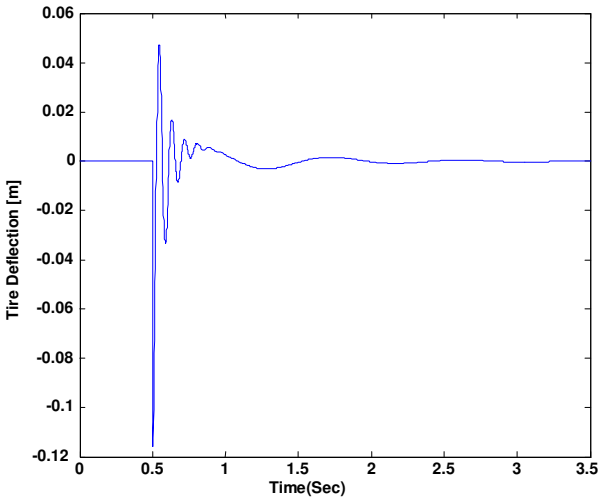

Fig10. Displacement of the center of wheel group mass center

In the first simulation an isolated bump on a flat surface road is used to shock the suspension system. The displacement due to road surface can be represented as follow [13]:

$$z_r(t)=\begin{cases}\frac{a}{2}(1-\cos(\frac{2\pi\upsilon_0}{\ell}t)) & 0\le t\le\frac{\ell}{\upsilon_0}\\ 0 & t>\frac{\ell}{\upsilon_0}\end{cases} \quad (15)$$

where a and ℓ are the height and length of the bump and υ_0 is the velocity of horizontal movement of the car. In the performed simulations in this stage the above parameters are considered as: $a=0.15\ m$, $\ell=5\ m$, and $\upsilon_0=60\ km/h$. The curve is sinusoidal wave form with height of 15 cm, where its width is related to the horizontal velocity of the car. This curve is applied to the robot two times. The acceleration of the car chassis is shown in Fig. 11 to check comforting of the passenger. Figure 12 shows the displacement of chassis mass center relative to the ground surface. The turbulence of the chassis mass center will continue for about 3 seconds. The peak of displacement of chassis mass center of the car is also about 12.5 cm which is less than the peak of road bumps (15 cm). Deviation of suspension length is also shown in Fig. 13. The highest allowable changes in the length of suspension part for the selected system is $z_{\max}=0.08\ m$. As we can see in the Fig. 13, the maximum deviation of suspension part (about 11 to 12.5 cm) is higher than allowable range, so the suspension system has encountered to a problem for isolated bump and transfers heavy shock to the spring and shock absorber. Note that since the spring coefficient of the wheel group is high, it is expected that the displacement of the road surface is directly transferred to the wheel mass center.

The ability of the car to maintain on the road, can be checked by comparing the dynamic and static loads imposed on the tire. The static load can be calculated through $F_s=g(m_s+m_u)$, which implies that $F_s=3528\ N$. The amount of the dynamic load, calculated through $F_d=k_t[z_u(t)-z_r(t)]$, is shown in Fig. 14. It shows that for isolated bumps with the initial speed of $60\,km/h$, the tire does not lose its contact with the road surface.

In the next step, an uneven bump on flat surface road is used to inspect the reaction of suspension system to this shock. For this purpose, the following equation is considered for the displacement of the road [13]:

$$z_r(t)=\begin{cases}a & 0.2\le t\le 0.2+\frac{\ell}{\upsilon_0}\\ -a & 3.2\le t\le 3.2+\frac{\ell}{\upsilon_0}\\ 0 & otherwise\end{cases} \quad (16)$$

where, a and ℓ are height and length of the bump and υ_0 is the horizontal speed of the car. In our simulation, the parameters are considered as before. Since the simulation results shows the similar behaviors, it is ignored to show them because of increasing the number of figures. Just the curve of dynamic load on the tire is shown in the Fig. 15.

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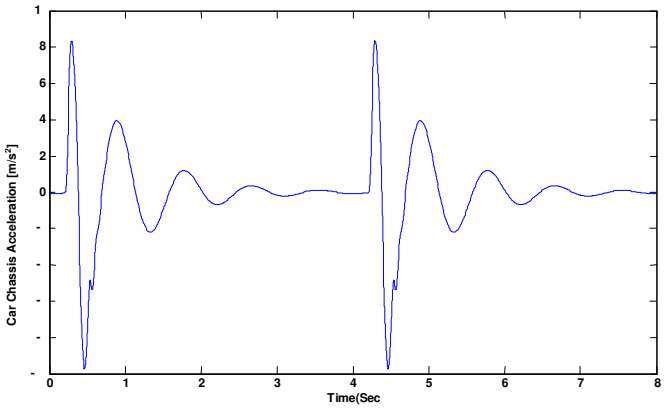

Fig11. The acceleration of the car chassis for isolated bumps

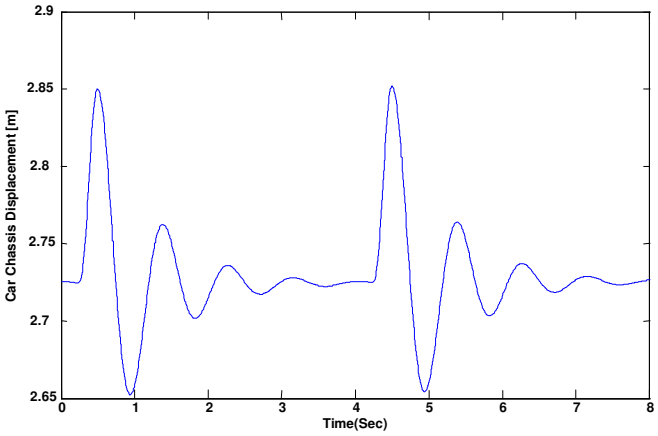

Fig12. The displacement of chassis mass center for isolated bumps

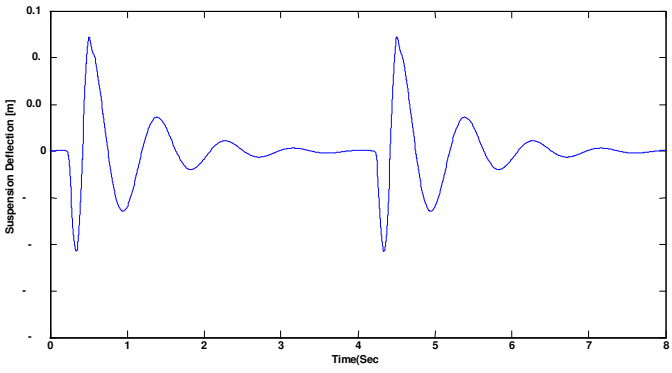

Fig13. Deviation of suspension length for isolated bumps

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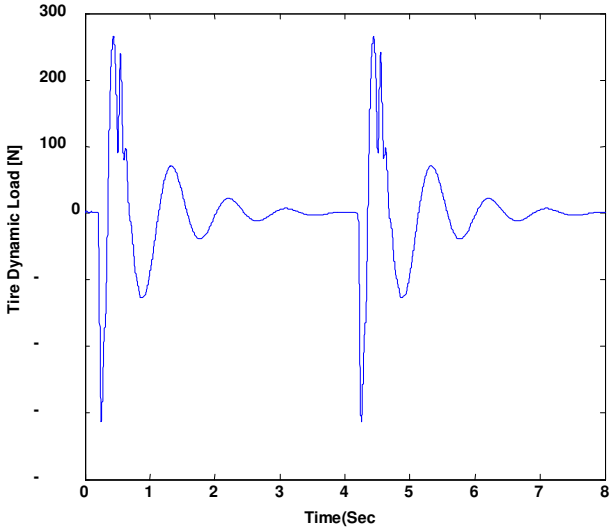

Fig14. Dynamic load on the tire for isolated bumps

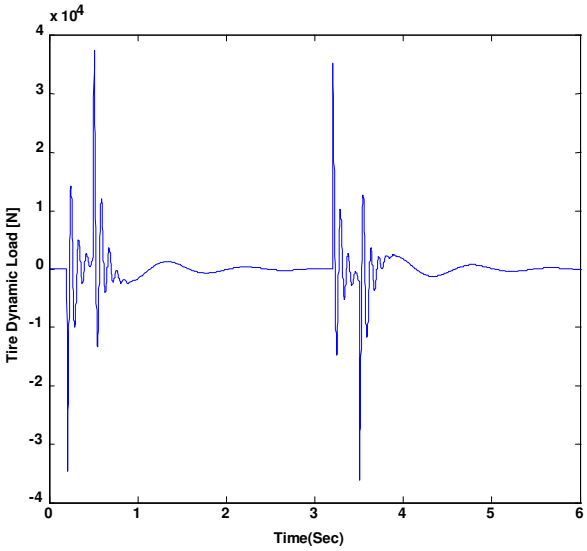

Fig15. Dynamic load on the tire for uneven bump

It shows that, in this case, the tire of the car lose its contact with the road surface for a short while.

5. Conclusion

This paper presented a new mechanism for inspecting the suspension system of the vehicles using parallel robots so called Stewart. It was simulated over a quarter model of a car and the reactions of the suspension system such as tire, spring and shock absorber were studied. A PID controller was used to control and guidance of the Stewart robot. It was shown that any road deflection can be simulated by imposing a suitable trajectory to the robot. The suggested suspension system was evaluated by applying isolated bumps and uneven bumps on flat surface road. Some operation criteria such as comforting of passenger and remaining of the vehicle on the road, were investigated for different road turbulences. The simulation results show a satisfactory accuracy and applicability for the proposed system compared to the commonly used systems. In the most commonly used systems there are four hydraulic Jacks with mobility just in vertical direction with low speed and low accuracy, while with the proposed system is able to test the

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suspension systems against to any force in any direction with high speed and high accuracy.

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